

Supersymmetric rings in field theory

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ABSTRACT: We study the dynamics of BPS string-like objects obtained by lifting monopole and dyon solutions of $N = 2$ Super-Yang-Mills theory to five dimensions. We present exact traveling wave solutions which preserve half of the supersymmetries. Upon compactification this leads to macroscopic BPS rings in four dimensions in field theory. Due to the fact that the strings effectively move in six dimensions the same procedure can also be used to obtain rings in five dimensions by using the hidden dimension.

KEYWORDS: Supersymmetric gauge theory, Solitons Monopoles and Instantons, String theory and cosmic strings.

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1. Introduction

In recent years many phenomena of D -brane physics have found a very similar implementation in the more conventional arena of supersymmetric field theories so that it is natural to consider similar constructions in this context. In this note we study traveling wave solutions on solitonic strings and use them to build BPS rings in field theory. Our starting point will be pure $N = 2$ Super-Yang-Mills (SYM) in four dimensions. This celebrated theory has BPS monopoles and instantons (for reviews see [1, 2]). By lifting these solutions to higher dimensions one obtains BPS strings in five and six dimensions. We consider traveling waves propagating on these strings and present the exact field theory description of these configurations. With the explicit wave solutions at our disposal we can then build stationary rings in one lower dimension by compactifying the theory on a circle. The rings appear as the lower dimensional projection of the string profile traveling in the extra-dimension. The excitations of the string probe the extra dimension so in this regard these configurations are not solutions of the lower dimensional theory.

At first sight it might look surprising that we find supersymmetric rings in $4D$ given the fact that a closed loop can only carry a scalar central charge and the only BPS particle solutions of $N = 2$ SYM are monopoles and dyons. The resolution to this is simply that the rings are intrinsically higher dimensional objects. From the four dimensional point of view they carry a monopole charge and the extra-dimensional momentum which contribute to the same central charge in the supersymmetric algebra. For a given monopole charge

the loop can have any size depending on the momentum flowing in the extra-dimension. Similar arguments hold for the instanton strings in $6D$. In $5D$ we also study non-relativistic strings obtained from dyon solutions and their generalizations. The waves travel at a speed smaller than the speed of light in this case.

The paper is organized as follows. In section 2 we provide the general strategy to construct static ring solutions in d dimensions starting from an infinite string in $d + 1$ dimensions. In section 2.1 we present a first example of this procedure in the Abelian Higgs model. In 2.2 we comment on the generalization of our construction to account for multiple winding strings in the context of σ -model strings. In section 3 we consider in detail the string obtained by lifting the monopole solution of $N = 2$ SYM and present the general traveling wave solution. Section 3.1 discusses the BPS properties of the traveling waves. In 4 we analyze the dyonic strings and their excitations. In 4.1 we show how ring-like solutions can be obtained even without the presence of a real extra-dimension when the string has extra-zero modes besides the position. We conclude in section 5. In the appendix we generalize these results to the instanton strings in six dimensions.

2. Construction

In this section we describe the general method to construct the static loop configurations (rings) from solitonic strings propagating in one more dimension. The first step in our construction is to obtain the field theory solutions for the string-like objects. This can be done, for objects of different co-dimension, by first identifying the appropriate field theories that possess point-like solitonic solutions, such as vortices in $2 + 1$ dimensions, monopoles in $3 + 1$, etc. . . and then uplift those solutions to one more dimension by considering the field theory configurations invariant along the extra-dimension.

In general, the solutions in the lower dimensional theory depend on a set of integration constants. These integration constants parameterize the massless zero modes of the $1 + 1$ low energy effective action. There could be several such zero modes associated with different aspects of the solution as its position, size, orientation in field space. An excitation of the zero modes corresponds to a traveling wave. For our purposes we mainly focus on the zero modes corresponding to the position of the string. If the object is Lorentz invariant along its extended direction, i.e its energy momentum tensor is that of a relativistic string, geometrical arguments imply that the zero modes corresponding to the position of the string are governed by the standard Nambu-Goto action,

$$S = -T \int d\tau d\sigma \sqrt{-\gamma} \tag{2.1}$$

where T denotes the tension of the string, (τ, σ) are the world-sheet coordinates and γ is the determinant of the induced metric on the string γ_{ab} , where a and b denote the internal indexes on the worldsheet.

Solutions of the equations of motion derived from this Nambu-Goto action have been known for a long time, and it is straightforward to find solutions describing traveling waves moving at the speed of light along an infinite straight string [3]. In the static gauge $X^0 = \tau$,

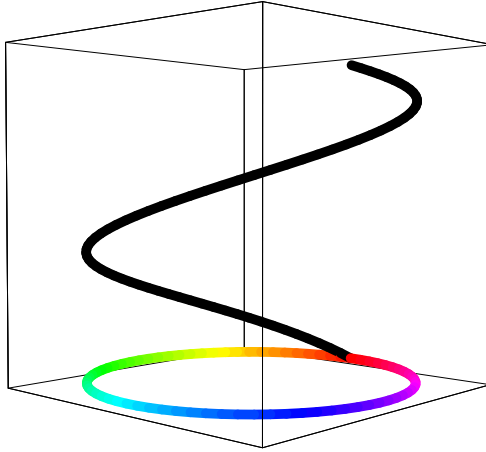


Figure 1: Snapshot of the string configuration and its ring projection in $2 + 1$ dimensions.

$X^d = \sigma$, these solutions take the form,

$$\begin{aligned} X^0 &= \tau \\ X^i &= \psi^i(\sigma \pm \tau) \\ X^d &= \sigma . \end{aligned} \tag{2.2}$$

The functions $\psi^i(\sigma \pm \tau)$ are arbitrary so that the wave can have any shape provided that the excitation travels in only one direction. These type of solutions have special interest in string theory [4–9].

Let's now consider the significance of these solutions on a compactified space-time of the form $M_d \times S^1$. A straight string wrapping around the extra-dimension would be seen from d -dimensions as a point particle. On the other hand, if we allow the string to carry a wave of the position zero mode, the profile of the string will appear to the low energy observer as a closed loop of the string. This can be achieved in the scheme described above by imposing that the functions ψ^i have the same periodicity as the extra dimension ¹,

$$\psi^i(z) = \psi^i(z + 2\pi R) \tag{2.3}$$

in which case the solution (2.2) is also solution of the theory on $M_d \times S^1$ where z parametrizes the angular coordinate along the circle, i.e. $0 < z < 2\pi R$. The energy of the configuration takes the simple form,

$$E = T \int_0^{2\pi R} d\sigma (1 + \vec{\psi}'^2). \tag{2.4}$$

This is just the sum of the energy of the unperturbed string and the momentum carried by the wave in the extra-dimension. Since for a macroscopic ring the amplitude of the fluctuation must be larger than the radius of the extra dimensions, it follows from this equation that the energy of the configuration is dominated by the momentum.

¹More in general one can consider strings winding k times around the S^1 in which case the constraint becomes $\psi^i(z) = \psi^i(z + 2k\pi R)$.

The fact that the lower dimensional observer detects a static loop might be puzzling at first because loops of relativistic strings should contract under the effect of their tension. The resolution to this puzzle is very simple. The low energy observer would not be able to probe the position of the string in the extra-dimensional space, but would definitely see their effect on the motion of the string because the effective action of the string is now modified by the presence of the scalar field $\chi(\sigma, \tau)$ that parameterizes the position of the string along the extra-dimensional circle, S^1 , namely,

$$S_{eff} = -T \int d^2\xi \sqrt{-\gamma_{d+1}} = -T \int d^2\xi \sqrt{-\gamma} \sqrt{1 - \gamma^{ab} \partial_a \chi \partial_b \chi}. \quad (2.5)$$

This type of effective action for a string propagating in four dimensions was originally studied in the context of dimensional reduction by several authors [10], and it may be regarded as a particular example of a much broader type of models where the strings can have new degrees of freedom propagating on the worldsheet, the superconducting string models [11]².

These new degrees of freedom affect the mechanical properties of the string by reducing its local effective tension thus allowing the existence of static closed loop configurations [12, 13]³. The stability of the string can be understood either from the lower dimensional point of view by the angular momentum created by the induced current along the string [13] or from the higher dimensional perspective as the angular momentum carried by the travelling wave in the plane orthogonal to the propagation of the wave [6, 8].

In the rest of the paper we would like to show how solutions of the type described above can be realized as smooth solitonic objects in ordinary field theory. In fact we will find that these solutions obtained in the thin-limit can be lifted to solutions of the full field theory equations for arbitrary large fluctuations.

2.1 Abelian-Higgs model strings.

In order to see how the construction outlined before works in field theory, we will first consider the Abelian-Higgs model in 3 + 1 dimensions,

$$S_{AH} = \int d^4x \left[|D_\mu \phi|^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{e^2}{2} (|\phi|^2 - \eta^2)^2 \right] \quad (2.6)$$

where, as usual, we define, $D_\mu \phi = (\partial_\mu - ieA_\mu)\phi$ and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. The constants have been chosen so that this action is the bosonic part of a supersymmetric theory. As is well known this theory admits solutions which satisfy first order Bogomol'nyi equations describing a vortex in the $x - y$ plane. These solutions preserve half of the supersymmetries and are characterized by the quantized magnetic flux. As shown in a nice paper by Vachaspati and Vachaspati [15], it is possible to find the general solution of an arbitrary traveling wave on a straight string. The key point of that paper was to identify the precise modifications that one would have to introduce to the ansatz in order to solve the full

²Here we used the word superconducting to refer to all the string models with extra degree of freedom even if their currents are not coupled to any electromagnetic field.

³These stable loops are known in the cosmic string literature as 'vortons'. See [14] and references therein.

nonlinear equations of motion. They then showed that the ansatz,

$$\begin{aligned}
 \phi &= \Phi(X, Y) \\
 A_x &= \mathcal{A}_x(X, Y) \\
 A_y &= \mathcal{A}_y(X, Y) \\
 A_t &= -\psi'_x \mathcal{A}_x(X, Y) - \psi'_y \mathcal{A}_y(X, Y) \\
 A_z &= \pm A_t
 \end{aligned}
 \tag{2.7}$$

solves the equations of motion for any Φ and \mathcal{A}_i which satisfy the static Bogomol'nyi equations. The solution depends on $X = x - \psi_x(z \pm t)$ and $Y = y - \psi_y(z \pm t)$ which denote the shifted time dependent positions of the string in the $x - y$ plane for two arbitrary functions ψ_x and ψ_y . It can be shown by considering the variations of the fermions that complete the supersymmetric theory that the traveling waves preserve or break the supersymmetries of the background depending on the direction of the wave. We will explain this fact in section 3.1.

The wave (2.7) is the field theory realization of the Nambu-Goto solution (2.2). We can also compute the energy of this configuration at the field theory level which agrees with (2.4) showing that the dynamics of the traveling waves is exactly captured by the Nambu-Goto approximation.

2.2 Multiple winding solutions. (σ -model Strings)

In the string theory examples of refs. [5, 4, 6], configurations with multi-strand strings or multiple winding strings were also considered. In principle the same could be done for the field theory strings. Generically if the field theory is supersymmetric there exist BPS multi-center solutions and so there will be configurations with independent waves for each of these strings. Unfortunately for the gauge strings of the Abelian-Higgs model (or of SYM considered in the following sections) the wave ansatz (2.7) does not generalize in an obvious way to multi-center solutions. Here we consider the same problem in the context of σ -model strings in 3 + 1 dimensions and show that in that case multi-center waves can easily be found.

It is well known that non-linear σ -models with Kähler target space admit static string configurations [16]. These solutions are classified by a topological number and the energy of the configuration, in that topological sector, is bounded by its charge. This is due to the fact that this model can be embedded in a supersymmetric model whose target manifold is Kähler. Here for simplicity we will investigate a particularly simple model, the CP^1 model. The advantage of using this simple model is that we will obtain an explicit expression for the multiple winding string by making use of the well known multiple center solutions but let us remark that a similar construction will work in general. For the CP^1 model the action is given by,

$$S = \int d^4x \frac{\partial_\mu \phi \partial^\mu \bar{\phi}}{(1 + \phi \bar{\phi})^2}
 \tag{2.8}$$

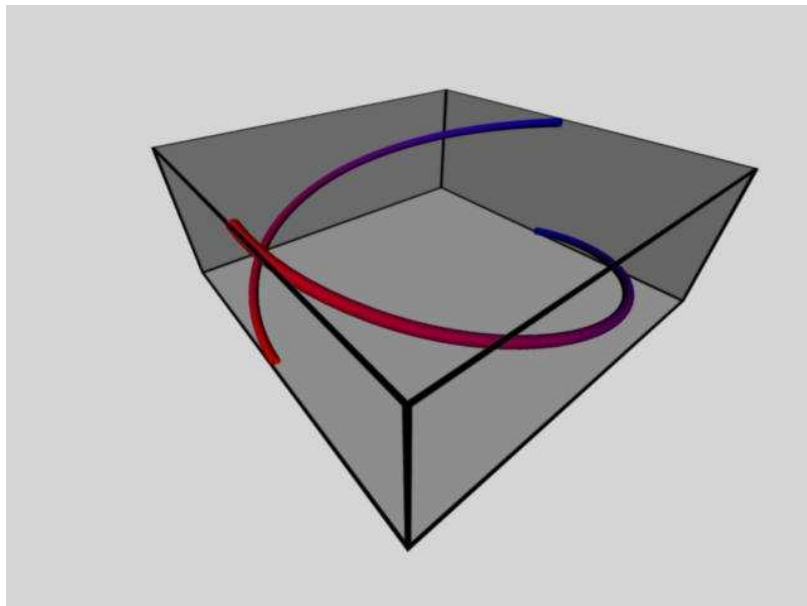


Figure 2: Snapshot of the multiwinding string configuration.

from which we obtain the equation of motion,

$$\partial_\mu \partial^\mu \phi - \frac{2\bar{\phi} \partial_\mu \phi \partial^\mu \phi}{1 + \phi\bar{\phi}} = 0. \tag{2.9}$$

We are interested in static string-like solutions of this equation where the fields only depend on the x and y coordinates. The equation above is satisfied by any holomorphic function $\phi(x + iy)$. An isolated string of unit topological charge centered at zero corresponds to,

$$\phi(x + iy) = \frac{1}{x + iy}. \tag{2.10}$$

Much in the same way as in the Abelian-Higgs model we obtain a traveling wave solution by taking,

$$\phi(x, y) = \frac{1}{((x - \psi_x(z \pm t) + i(y - \psi_y(z \pm t)))}. \tag{2.11}$$

This configuration is therefore another field theory realization of the string wave characterized, in the thin wall approximation, by the solution (2.2). The simplicity of the CP^1 model allows us to write simple solutions of higher topological charge, such as,

$$\phi_2(x, y) = \frac{1}{((x - 1) + iy)((x + 1) + iy)} \tag{2.12}$$

which represents two strings of unit charge located at $(x = 1, 0)$ and $(x = -1, 0)$. We can now wonder what kind of configurations we will obtain upon compactification by exciting traveling waves on the different strings. It is clear that we could get a collection of “disconnected” rings in $2 + 1$ dimensions, each of which associated with a different string. There is however a more interesting possibility of constructing a single ring by imposing

the correct periodicity on each of the strings in such a way that the end result becomes a single string of higher winding along the extra dimension. A simple example of this can be obtained by using the previous multicenter solution (2.12) and taking for simplicity a sinusoidal traveling wave,

$$\phi_2(x, y) = \frac{1}{((x - \cos(z \pm t)) + i(y - \sin(z \pm t)))(x + \cos(z \pm t)) + i(y + \sin(z \pm t))}. \quad (2.13)$$

This configuration describes a couple of helicoidal strings propagating along the z direction. By requiring the z direction to be compact with period π we can identify the previous solution as a single string winding two times along the extra dimension before closing on itself (See figure (2)). On the other hand, it is also possible to find this kind of string state as a solution of the effective action (2.5),

$$X^0 = \tau \quad (2.14)$$

$$X^1 = \cos(\sigma \pm \tau) \quad (2.15)$$

$$X^2 = \sin(\sigma \pm \tau) \quad (2.15)$$

$$\chi = \sigma$$

where $0 < \sigma < 2\pi$ and therefore, the field χ winds two times as we move along string loop. It is straightforward to extend the solution found above to more general wave forms as well as to higher winding number.

We would also like to note that, as in many of the cases we will talk about in this paper, there are other zero modes on the low energy effective theory of these strings. Exciting some of those along the compactified direction will lead to different type of ring-like configurations in the lower dimensional space-time. An example of such situation is the Q-lump solution in the CP^1 model [17, 18].

3. Monopole strings in 5D

Let us now see how we can obtain ring solutions in 3 + 1 dimensions. Following the procedure described in the previous section, we should first find a model that possesses as part of its spectrum string-like objects in 4 + 1 dimensions. The simplest supersymmetric theory of this kind corresponds to $SU(N)$ Super-Yang-Mills theory in 4 + 1 dimensions. The bosonic part of the action is given by,

$$S_{YMH} = \int d^5x \text{Tr} \left[-\frac{1}{2} F_{MN} F^{MN} + D_M \phi D^M \phi \right] \quad (3.1)$$

with

$$F_{MN} = \partial_M A_N - \partial_N A_M + ie[A_M, A_N] \quad (3.2)$$

$$D_M \phi = \partial_M \phi + ie[A_M, \phi],$$

where ϕ and A_M are valued in the algebra of $SU(N)$. Upon KK reduction to four dimensions the theory reduces to pure $N = 2$ SYM. As well known, this theory admits

supersymmetric monopole solutions. These configurations correspond to solutions of the first order equations,

$$B_i = \pm D_i \phi \tag{3.3}$$

where $B_i = -1/2 \epsilon_{ijk} F_{jk}$. The solutions saturate a Bogomol'nyi bound on the energy where the energy is equal to the magnetic charge in appropriate units. For simplicity of notation in the rest of the paper we will restrict our analysis to the case where the gauge group is SU(2) but the generalization to SU(N) is immediate. For the case of monopole charge one, the explicit solution of the Bogomol'nyi equations is given by,

$$\begin{aligned} \phi &= \Phi = \frac{x^a \sigma^a}{2er^2} H(ver) \\ A_i &= \mathcal{A}_i = -\epsilon_{aij} \frac{x^j \sigma^a}{2er^2} (K(ver) - 1) \\ A_0 &= A_4 = 0, \end{aligned} \tag{3.4}$$

where r is the 4D radial direction, v the asymptotic value of ϕ , σ^a are Pauli matrices and

$$\begin{aligned} H(y) &= y \coth(y) - 1 \\ K(y) &= \frac{y}{\sinh(y)}. \end{aligned} \tag{3.5}$$

The monopole solutions written above obviously solve the full five dimensional equations of motion since nothing depends on the fifth coordinate. In five dimensions these solutions become string-like configurations where each section of the string is given by the monopole solution. Their tension is given by,

$$T = \frac{4\pi v}{e}, \tag{3.6}$$

where to agree with the 5D normalizations v and e have mass dimensions 3/2 and -1/2 respectively.

Having found smooth strings in 5D we now wish to consider waves traveling on these objects, i.e. excitations of the zero modes living on the string. In particular to construct rings we are interested in the waves of the position of the string. The five dimensional equations of motion that follow from 3.1 are given by,

$$\begin{aligned} D_M F^{MN} &= ie [\phi, D^N \phi], \\ D^M D_M \phi &= 0. \end{aligned} \tag{3.7}$$

In order to find the exact field theory description for the traveling waves on the monopole-string we closely follow the example of the Abelian-Higgs model presented previously. Promoting the string position to a light-like field the equation of motion for the scalar and the components of the gauge field equations transverse to the string are automatically satisfied as long as $A_0 = \pm A_4$. These can be chosen so that also the 0 and 4 equations for the gauge field are solved. In fact, despite of the non-abelian nature of the monopoles, the straightforward generalization of the abelian case works for the monopole-strings as well. The most general solution representing a transverse traveling wave moving at the speed of

light along the x^4 direction is given by,

$$\begin{aligned}
 \phi &= \Phi(X, Y, Z) \\
 A_i &= \mathcal{A}_i(X, Y, Z) \quad i = 1, 2, 3, \\
 A_0 &= -\sum_{i=1}^3 \psi'_i \mathcal{A}_i \\
 A_4 &= \pm A_0
 \end{aligned} \tag{3.8}$$

where $\psi = \psi_i(x^4 \pm x^0)$ is the three dimensional vector that parameterizes the arbitrary transverse perturbation, and $(X, Y, Z) = (x_i - \psi_i(x^4 \pm x^0))$ are the shifted position of the string core due to the presence of the fluctuation. This solution actually holds not only for the one monopole solution (3.4) but for any solution of the Bogomol'nyi equations (3.3). In the more general case of the multi-center solutions there will be traveling waves associated with the motion of each individual string. The solution (3.8) describes the excitation corresponding to the shift of the center of mass of the system.

Beside the zero modes corresponding to the position, the monopole-string has yet another bosonic collective coordinate χ associated to the global U(1) transformation (see [1] for the details). There are traveling waves solutions for this mode as well, as for any other zero mode that lives on the string such as fermionic zero modes required by supersymmetry⁴. The traveling wave solution is given in this case by,

$$\begin{aligned}
 \phi &= \Phi \\
 A_0 &= \chi' \Phi \\
 A_i &= \mathcal{A}_i \\
 A_4 &= \pm A_0
 \end{aligned} \tag{3.9}$$

where χ is an arbitrary function of $(x^4 \pm x^0)$. The χ -wave can be added to the waves of the positions giving the most general bosonic traveling wave on the monopole string which depends on four arbitrary functions. From this solution we can see that χ appears on the same footing as the other fluctuations, if we interpret the scalar field as the fifth component of a gauge field in six dimensions. Actually this is not an accident and as we will see in section 4 it follows from the fact the monopole string effectively moves in six dimensions. As a special case of (3.9), we can take χ to be linear in $(x^4 \pm x^0)$. With this choice we obtain a static configuration translationally invariant along the world-volume (this would not be possible for the wave of the position). Since these solutions do not depend on on x_4 they are also a solutions of SYM in $4D$; in fact they are contained within the general solutions found in [20]. From the $5D$ point of view these solutions carry linear momentum and reproduce the ones found in [21]. Note, however that our solutions in 3.9 are more general than the ones obtained in the literature since they hold for any function $\chi(x^4 \pm x^0)$, but those depend explicitly on x^4 so they are not solutions of the lower dimensional theory.

⁴Stable monopole string rings may also be obtained by exciting fermionic waves on the string in a similar way to the configurations recently proposed in [19].

3.1 Supersymmetry

We now examine the traveling wave solutions found in the previous section from the supersymmetry point of view (see also [22]). As we will show most of the properties can be derived in a model independent way from the supersymmetry algebra. Very similar arguments hold for the Abelian-Higgs model vortices and σ -model strings considered in the section 2.

The supersymmetric extension of the action (3.1) is invariant under $N = 1$ supersymmetry in $5D$ which corresponds to $N = 2$ supersymmetry in $4D$. The variation of the fermions under supersymmetry is,

$$\delta\psi = (\gamma^{MN} F_{MN} - \gamma^M D_M \phi) \epsilon \tag{3.10}$$

where ϵ is a four component Dirac spinor. The straight string is just the lift to $5D$ of the $4D$ BPS monopoles where all the fields are independent from the fifth coordinate. It follows immediately that this state is supersymmetric. What is perhaps less obvious but still true is that the waves traveling at the speed of light on the string can also be BPS. Using the explicit expressions (3.8) in the fermion variations (3.10), it is possible to check that these solutions are one half BPS as the background if the wave travels in the direction specified by the monopole charge (i.e. upward for positive monopole charge in our conventions). Waves traveling in the opposite direction break all the supersymmetries instead. The ring configuration are obtained as a special case of traveling wave solutions so they are also supersymmetric as long as the wave travels in the appropriate direction. From the lower dimensional point of view these objects carry a scalar central charge as the magnetic monopoles. More precisely the central charge contains two contributions, one is the magnetic charge and the other the momentum flowing in the extra-dimension.

We would now like to show that the BPS properties of the traveling wave are a direct consequence of the supersymmetry algebra in $5D$. This algebra is formally identical to the $N = 2$ supersymmetry algebra in $4D$ which in Weyl notation is given by,

$$\begin{aligned} \{Q_\alpha^A, \bar{Q}_{\dot{\alpha}B}\} &= \sigma_{\alpha\dot{\alpha}}^\mu P_\mu \delta_B^A \\ \{Q_\alpha^A, Q_\beta^B\} &= \epsilon_{\alpha\beta} \epsilon^{AB} Z. \end{aligned} \tag{3.11}$$

In $4D$ Z is a complex central charge carried by BPS point-like objects such as monopoles and dyons. With this parametrization, in $5D$ the imaginary part of Z represents the fifth component of the momentum so that the algebra has single real central charge. A string can both carry a scalar central charge or a vectorial central charge (the algebra also contains a triplet of 2-index central charges but this cannot be carried by a string). Considering the Lorentz transformations of the charges it follows that an object Lorentz invariant along the world-volume direction can only carry a vectorial central charge. Since monopole strings are manifestly Lorentz invariant it follows that the monopole charge of the string enters in the algebra as “momentum” since P^M is the only vectorial charge in the $5D$ algebra. Explicitly the topological charge of the string in $5D$ is the vector,

$$P_{mag}^M = \epsilon^{0MNPQ} \int d^4x Tr[F_{NP} D_Q \phi]. \tag{3.12}$$

For an object carrying P^4 charge, the algebra (3.11) implies the bound on the energy,

$$E \geq |P^4|. \tag{3.13}$$

The formula is identical to the one of massless particles but now to P^M contributes the linear momentum as well as the magnetic charge (3.12). When the bound is saturated, as for the monopole strings, the object preserves half of the supersymmetries. Let us now add fluctuations of the zero modes living on the string moving in one direction. Since the excitation moves at the speed of light in space-time its energy must be equal to momentum. To see this more explicitly we can compute the energy momentum tensor,

$$T_{MN} = Tr \left[\frac{1}{4} g_{MN} F_{PQ} F^{PQ} - F_{MP} F_N^P - \frac{1}{2} g_{MN} (D_P \phi)^2 + D_M \phi D_N \phi \right]. \tag{3.14}$$

Using the solution (3.8) one finds,

$$\delta T_{00} = |T_{04}| = Tr \left(\sum_{i=1}^3 \psi'_i D_i \phi \right) + Tr \left(\sum_{i,j,k=1}^3 F_{ij} F_{ik} \psi'_k \psi'_j \right) \tag{3.15}$$

where δT_{00} denotes the difference of energy densities between the excited and the background string solutions (in the previous expression all the fields are evaluated at the shifted point $(x_i - \psi(x^4 \pm x^0))$). By integrating over the transverse directions the total energy reduces exactly to eq. (2.4) obtained from the Nambu-Goto action. Therefore the total charge appearing in the algebra is given by,

$$P^4 = P_{mag}^4 + P_{lin}^4$$

where,

$$P_{mag}^4 = 2\pi R \frac{4\pi v}{e}$$

$$P_{lin}^4 = \pm \frac{4\pi v}{e} \int_0^{2\pi R} dx^4 \sum_i \psi_i'^2 \tag{3.16}$$

The total energy is given by $E = |P_{mag}^4| + |P_{lin}^4|$. From the BPS bound (3.13) we derive that the vibrating string is BPS only if P_{mag}^4 and P_{lin}^4 have the same sign. When this condition is satisfied the traveling wave leaves unbroken the same supersymmetries of the background, it is invariant under four supercharges. Note that these arguments do not depend on the type of fluctuation as long as it travels at the speed light in the direction specified by the orientation of the string. It is useful to consider these facts from point of view of the 1+1 dimensional world-volume theory. Consider the background string solution. Projecting the 5D algebra on the subspace of the unbroken generators one finds that the unbroken supersymmetries form a (0,4) two dimensional superalgebra. This means that the supersymmetries act only on the movers in one direction, the string is chiral. As a consequence a massless excitation in the opposite direction leaves unbroken four supersymmetries. Let us consider the fermions. The zero modes of the monopole solution are four

bosonic coordinates and four fermions which have a definite chirality. Upon lifting the solution to five dimensions there will be four chiral fermions moving in only *one* direction and four bosons moving in *both* directions. The fermions are the goldstinos for partial breaking of supersymmetry and so they transform in the bosons under the action of the unbroken supersymmetries. As a consequence a wave of the fermions breaks all the supersymmetries. If however there are additional fermions on the string moving in the appropriate direction a wave of the fermions will also be BPS.

The scenario with fermions moving in both directions is automatically realized in $N = 4$ SYM. The previous analysis can be extended to this case with minor modifications (the algebra now contains a triplet vectorial central charge which is carried by the string). The background monopole string solution is identical to the one considered before. In this case the unbroken supersymmetries form a $(4, 4)$ two dimensional superalgebra. On the monopole string there are four fermions traveling in each direction which match the number of bosonic zero modes so the string is non-chiral. From this we can conclude that any wave is a quarter BPS state preserving the supercharges with opposite chirality with respect to the wave.

4. Non relativistic strings

The $5D$ SYM admits several other BPS strings. The properties of these objects however are rather different from the monopole strings since as we will show generically they break Lorentz invariance along the string.

One type of string-like object can be obtained by lifting the dyon solution of $4D$ SYM to $5D$. As shown by Mueller long ago [23], the dyon solution is related to the monopole solution by a boost. For the monopole string which is supported by ϕ this can be achieved interpreting ϕ as the fifth component of a gauge field in six dimensions and boosting in that direction⁵. The solution is given by,

$$\begin{aligned} \phi &= \Phi \left(\frac{x^a}{\cosh \theta} \right) \\ A_0 &= \tanh \theta \Phi \left(\frac{x^a}{\cosh \theta} \right) \\ A_i &= \frac{1}{\cosh \theta} \mathcal{A}_i \left(\frac{x^a}{\cosh \theta} \right) \\ A_4 &= 0 \end{aligned} \tag{4.1}$$

where Φ and \mathcal{A}_i are the monopole string solution (3.4). Note that the fields have been rescaled so that they approach the same vacuum. The energy per unit length is given by the dyon formula,

$$\rho = v \sqrt{Q_e^2 + Q_m^2} \tag{4.2}$$

where,

$$Q_e = Q_m \sinh \theta. \tag{4.3}$$

⁵This procedure can be made explicit by lifting the monopole string to $6D$ SYM where it becomes a supersymmetric (but non-relativistic) membrane.

The energy density is identical to the one for the (p, q) strings of type IIB string theory. In fact the formula just depends on the structure of the supersymmetry algebra which turns out to be similar in the two cases. There is however an important difference. From the point of view of the algebra in $5D$ the magnetic charge is the “momentum” charge while the electric charge contributes to the scalar central charge of the supersymmetric algebra,

$$X_{el} = v Q_e = 2 \int d^4x \text{Tr}[F_{0i} D_i \phi]. \tag{4.4}$$

The presence of the scalar central charge breaks Lorentz invariance along the string, i.e. the string is not relativistic. This also follows immediately from the solution since A_0 is different from zero.

Broken Lorentz invariance drastically alters the properties of the zero modes living on the string. In particular the zero modes will not travel at the speed of light. To see this explicitly one can compute the energy momentum tensor (3.14) for the dyon string. From the ratio of the pressure T_{44} to the energy density T_{00} (all other components of T_{MN} are equal to zero), we learn that the velocity of propagation on the string is given by,

$$v_s = \frac{Q_m}{\sqrt{Q_e^2 + Q_m^2}} = \frac{1}{\cosh \theta} \tag{4.5}$$

This formula can be directly obtained from the supersymmetry algebra by noticing that the only contribution to the tension arises from vectorial central charges.

The field theory waves on the dyonic strings cannot be easily found as for the monopole strings. The trick of boosting in the ϕ direction does not work in this case because for a traveling wave it would introduce a dependence on the extra coordinate. In fact since the speed of the traveling waves is smaller than the speed of light it is not even obvious at first that these excitations can be supersymmetric. We can however argue in the following way. As argued by Tong [2] the effective action for the monopole strings is the Nambu-Goto action of a string moving in one extra dimension χ which is compactified on a circle of radius $1/(ev)$,

$$S = -T \int d^2\sigma \sqrt{-\gamma_6} \tag{4.6}$$

where γ_6 is the determinant of the pull back of the six dimensional flat metric where $X_5 = \chi$. This action can be derived by considering the exact spectrum of BPS dyons in $4D$. A very important fact is that the action is Lorentz invariant in six dimensions, it effectively describes a relativistic string in $6D$. At least for the maximally supersymmetric case this string can be identified with the strings of the $(2, 0)$ superconformal field theory in $6D$ which is related to $5D$ SYM by compactification (see [2, 24] and refs. therein). From the Nambu-Goto point of view the dyon string corresponds to the monopole string moving with constant speed in the χ direction, $\chi = \beta t$. The constant β is related to the electric charge by,

$$\beta = \frac{Q_e}{\sqrt{Q_e^2 + Q_m^2}} \tag{4.7}$$

We can now consider waves propagating on the dyon background. From the Nambu-Goto action one finds that there are waves of arbitrary shape traveling at the reduced

velocity (4.5),

$$\begin{aligned}
 X^0 &= \tau \\
 X^i &= \psi^i(\sigma \pm v_s \tau) \\
 X^4 &= \sigma \\
 X^5 &= \beta \tau.
 \end{aligned}
 \tag{4.8}$$

This solution is a boost along the χ direction of the wave on the monopole string. The reduced velocity can then be interpreted as the red-shift due to the motion of the center of mass of the string in the hidden dimension. Since the effective action describes a supersymmetric string in six dimensions it is also clear that the traveling waves are supersymmetric (as long as they travel in the appropriate direction), because they are just the boost of the supersymmetric waves on the monopole strings. The existence of supersymmetric traveling wave solutions at the level of the Nambu-Goto action strongly suggest that these solutions will be supersymmetric in the full field theory realization.

4.1 Hidden Dimension

Once we realize that the monopole string is a six dimensional relativistic string many other solutions are immediately available at the level of the Nambu-Goto action. Many of these solutions have a field theory correspondent. Starting from the monopole string in six dimensions,

$$\begin{aligned}
 X^0 &= \tau \\
 X^i &= 0 \\
 X^4 &= \sigma \\
 X^5 &= 0,
 \end{aligned}
 \tag{4.9}$$

we can obtain new strings in $5D$ by boosting in the X^5 direction and by rotating in the plane $X^4 - X^5$. The most general solution of this kind is given by,

$$\begin{aligned}
 X^0 &= \tau \\
 X^i &= 0 \\
 X^4 &= \sigma \\
 X^5 &= \frac{\tanh \theta}{\cos \alpha} \tau - \tan \alpha \sigma,
 \end{aligned}
 \tag{4.10}$$

where we have reparametrized the world-sheet coordinates by choosing the static gauge along X^4 . The boost and rotation has a direct analog in field theory. Treating again ϕ as the fifth component of a gauge field in $6D$ we can obtain static strings in $5D$ by performing the same boost and rotation as in the Nambu-Goto case. The solution obtained by a rotation for example corresponds to lifting a $4D$ monopole solution which is supported by a linear combination of ϕ and A_5 while the boost corresponds to the dyon considered before. In order to find the field theory description of the solutions (4.10) we need however to require that they asymptote to the same vacuum (determined by the value of the scalar ϕ at infinity) as the monopole string. Following [23], this can be achieved by rescaling of

the coordinates. The general solution is then given by,

$$\begin{aligned}
 \phi &= \Phi \left(\frac{x^a}{\cosh \theta \cos \alpha} \right) \\
 A_0 &= \frac{\tanh \theta}{\cos \alpha} \Phi \left(\frac{x^a}{\cosh \theta \cos \alpha} \right) \\
 A_i &= \frac{1}{\cosh \theta \cos \alpha} \mathcal{A}_i \left(\frac{x^a}{\cosh \theta \cos \alpha} \right) \\
 A_4 &= \tan \alpha \Phi \left(\frac{x^a}{\cosh \theta \cos \alpha} \right).
 \end{aligned} \tag{4.11}$$

These strings have the same energy density and pressure as the one obtained from the Nambu-Goto solution (4.10) showing that they are the field theory realization of those solutions. The charges carried by these strings are in general,

$$\begin{aligned}
 P_{mag}^4 &= -\epsilon^{0ijk4} \int d^4x \text{Tr}[F_{ij} D_k \phi] \\
 X_{inst} &= -\epsilon^{0ijk4} \int d^4x \text{Tr}[F_{ij} D_k A_4] \\
 P_{el}^4 &= 2 \int d^4x \text{Tr}[F_{0i} D_i A^4] \\
 X_{el} &= 2 \int d^4x \text{Tr}[F_{0i} D_i \phi] \\
 P_{lin}^4 &= \int d^4x T_0^4
 \end{aligned} \tag{4.12}$$

where X and P^4 refer to the fact that they appear in the supersymmetric algebra as scalar or vector charge. Note that all the solutions (4.11) have magnetic monopole charge one. It is easy to see that a rotation of the monopole string induces an instantonic charge X_{inst} while a boost generates X_{el} . The composition of boosts and rotations generates also linear momentum so that the general solution (4.11) carry all the above charges. In the Nambu-Goto description these charges can be identified with the winding and momentum of the string. For example winding in the fifth dimension corresponds to instanton charge while momentum corresponds to electric charge.

For each of the strings (4.11) there are also traveling wave solutions. At the Nambu-Goto level these solutions are again the boost of the traveling waves on the monopole string. One finds that in this case the velocity of propagation is given,

$$v_s = \frac{\cos \alpha}{\cosh \theta} \mp \sin \alpha \tanh \theta \tag{4.13}$$

which agrees with the field theory computations. The \mp refers to the velocity of the perturbations in opposite directions which can be different due to the fact that in general the strings are not relativistic and carry momentum. Being Lorentz transformations of supersymmetric solutions these waves are supersymmetric at the Nambu-Goto level so we expect this property to continue in the field theory solutions.

If we take seriously the Nambu-Goto action we can also find point-like solution in five dimensions,

$$\begin{aligned} X^0 &= \tau \\ X^i &= 0 \\ X^4 &= 0 \\ X^5 &= k \sigma \end{aligned} \tag{4.14}$$

which describes a strings winding around the hidden dimension X^5 . Since the radius of this dimension is $1/(ev)$ one finds that the energy of this configuration is given by, [2],

$$M = k \left(\frac{2\pi}{ev} \right) T = \frac{8\pi^2 k}{e^2} \tag{4.15}$$

where k is the number of windings. Remarkably this formula exactly reproduces the mass of the instanton particles of the theory despite the fact the solution has no string structure in $5D$. These solutions are singular in field theory but can be considered as a limit of the dyonic instantons of [25] where the electric charge goes to zero. The W bosons on the other hand can be identified with the KK modes of the theory [24]. This suggests that the Nambu-Goto description might be exact for the BPS objects of the theory.

Following the same procedure as in the previous sections we obtain rings in $5D$ by exciting waves of the position now traveling in the hidden dimension. For example a circular ring lying in the $X^1 - X^2$ plane is represented by,

$$\begin{aligned} X^0 &= \tau \\ X^1 &= R \sin \left(n e v \left(\sigma \pm \frac{\tau}{k} \right) \right) \\ X^2 &= R \cos \left(n e v \left(\sigma \pm \frac{\tau}{k} \right) \right) \\ X^5 &= k \sigma, \end{aligned} \tag{4.16}$$

where n is the number of times the projection of the string in $5D$ goes around the circle in the $X^1 - X^2$ plane. The mass of the loop is given by,

$$M = \frac{8\pi^2 k}{e^2} + \frac{8\pi^2 n^2 v^2 R^2}{k}, \tag{4.17}$$

where the two contributions arise from the winding and the momentum flowing in the internal dimension which correspond to the electric and instantonic charges in (4.12). These are precisely the same charges carried by the dyonic instantons of SYM discussed by [25, 26, 21, 27]. These solutions, which are closely related to supertubes in string theory [28–30], carry instantonic and electric charge and their mass is given by,

$$M = X_{inst} \pm X_{el} \tag{4.18}$$

which saturates the BPS bound (for a recent discussion see [22]). We can therefore identify the $5D$ rings as some kind of dyonic instantons. To show this in more detail we can look

at a small element of the loop (4.16),

$$\begin{aligned} X^0 &= \tau \\ X^1 &= \sigma \\ X^5 &= \frac{k}{R n e v} \sigma \pm \tau. \end{aligned} \tag{4.19}$$

This has the form of the solution (4.10) which represents an excitation of the monopole-string we started from. We are then lead to interpret the dyonic instantons in field theory as the Nambu-Goto loops we have been discussing where the excitations along the string allow the possibility of having static string loops.

We should note that, since the thickness of the monopole string in comparable to the size of the hidden dimension, the Nambu-Goto description could break down. On the other hand, from the field theory realization (4.11), with the parameters given by (4.19), one can see that for a loop of radius R the thickness of the string constituent is given by,

$$\delta \sim \frac{nR}{k}, \tag{4.20}$$

which suggests that the description in terms of the Nambu-Goto solutions would be reliable for $k/n \gg 1$. In fact, already for $k = 2$ it was shown by [22] that dyonic instantons with string-like structure can be build. For the case $k = 1$ the only dyonic instantons known have spherical symmetry [25], so they cannot be immediately identified with the ring solutions of the Nambu-Goto action. It is however possible that, due to the BPS properties, even these solutions might be captured by the $6D$ Nambu-Goto action.

5. Conclusions

In general, any relativistic string admits wave solutions of arbitrary shape moving at the speed of light in space-time. These solutions can be used to construct stationary rings in one lower dimension since by choosing appropriately the periodicity of the fluctuations, the traveling waves are also solutions of the theory compactified on a circle. The ring is then the projection of the string profile on the lower dimensional space. While the low energy observer will only detect a static ring these solutions are truly higher dimensional since they involve excitations of the modes of the Kaluza-Klein tower.

In this paper we presented field theoretic realizations of the rings. In particular we considered the solitonic strings obtained by lifting the monopole solutions of $N = 2$ SYM in $4D$ to five dimensions (similar results hold for the instantonic strings in $6D$, see appendix). Generalizing previous work in the context of the Abelian Higgs model in $4D$ [15], we have derived the exact traveling wave solutions on these objects. These waves preserve one half of the supersymmetries if the wave travels in the direction determined by the charge of the monopole/instanton string. Therefore the rings are also BPS and can be thought as some kind of blown up monopoles.

The monopole strings studied in this paper are particularly rich because they effectively move in six dimensions. The presence of this extra degree of freedom allows us to construct

rings in five dimensions in an analogous way to superconducting string models and to supertubes in string theory. Moreover a constant motion in the hidden dimension induces electric charge on the string. The strings so obtained are just the lift of the dyon solution to $5D$ and they are supersymmetric but not relativistic. It would be very interesting to find a model with similar properties in four dimensions.

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A. Instanton strings

In this appendix we briefly consider the generalization of this work to the six dimensional case. In $4D$ beside supersymmetric monopoles and dyons, $N = 2$ SYM on the Coulomb branch also admits BPS instanton solutions. Many of the arguments used for the monopole strings can be repeated in this case as well. If we lift the theory to six dimensions the instantons become BPS strings invariant under four supersymmetries. In $6D$ the theory contains only a gauge field and a Dirac spinor in the adjoint representation of the gauge group so the lift of the $4D$ instanton is unique. The instanton string corresponds to a solution of the self-dual equations,

$$\begin{aligned} A_0 &= A_6 = 0 \\ F_{ij} &= \pm \star F_{ij}, \quad i, j = 1, 4 \end{aligned} \tag{A.1}$$

i.e. its transverse slices are identical to the instanton solution in $4D$. For the simplest case the solution to this equations can be written in singular gauge as,

$$A_i = \frac{\rho^2 x_i}{r^2(r^2 + \rho^2)} \eta_{ij}^a \sigma^i \tag{A.2}$$

where η^a is a triplet of 4×4 matrixes (see [2]).

This one instanton solution contains eight bosonic zero modes. Four of them are the position of the instanton-string, ρ determines its size and the remaining three modes correspond to the global gauge transformations (for the case of $SU(2)$). We now want to consider a wave propagating on the string. The string is Lorentz invariant so that the waves of the zero modes will propagate at the speed of light. As usual to find traveling waves we promote the integration constants of the static background to light-like fields. As for the monopoles strings the exact traveling wave solution has the form,⁶

$$A_i = \mathcal{A}_i(X, Y, Z, T)$$

⁶Traveling waves on instanton strings have also been discussed in [31].

$$\begin{aligned}
 A_0 &= - \sum_{i=1}^4 \psi'_i \mathcal{A}_i^a \\
 A_5 &= \pm A_0 .
 \end{aligned}
 \tag{A.3}$$

We can also build traveling wave solutions using the other zero modes living on the string. This is particularly simple for the size zero mode. In fact promoting ρ to an arbitrary function $\rho(x^5 \pm x^0)$ the equations of motion remain satisfied.

One can also excite the internal degrees of freedom of the SU(2) symmetry and based on the experience with the monopole strings these would also affect the mechanical properties of the strings, rendering them non-relativistic. In this regard it is possible that there are also stable static loops of the field theory instantonic strings much in the same way as the ones explained in section 4 for the monopole strings where the U(1) zero mode is embedded in the SU(2) case at hand. In fact, it can be shown that there are stable string configurations at the Nambu-Goto level when the internal manifold of the spacetime explored by the string is a S^2 sphere instead of a circle [32]. These solutions however will likely not be supersymmetric.

The solutions found above are all BPS as can be checked from the explicit supersymmetry variations. In 6D the minimal SUSY algebra contains a vector charge P_M and a triplet of tensorial central charges Z_{MN}^i . A static straight string pointing in the fifth direction carries the instanton charge which appears as fifth component of the momentum in the algebra. The solution is necessarily relativistic since there are no scalar central charges that can be carried by the string. By looking at the algebra of the unbroken supersymmetries one can see that as in the monopole case these form a (4, 0) superalgebra in 4D. Therefore, also in this case only waves traveling in one direction (determined by the sign of the instanton charge) will be supersymmetric.

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